#13 PRACTICAL INTERPRETATION & ESTIMATION 2.4

1. Estimate $P'(1940)$ and give a practical interpretation. $P$ represents the amount of carbon dioxide (ppm) in the atmosphere, $t$ represents the year.

$(1910, 290)$
$(1890, 270)$

$P(1940) = \frac{20}{20} = 1 \text{ ppm/year}$

**During 1940,**
**carbon dioxide increased by 1 ppm.**

![Graph showing carbon dioxide content from 1860 to 1960](image)

2. The speed of a car in mph can be expressed in terms of the length of a skid mark in feet when brakes are applied. Estimate $S'(20)$ and give a practical interpretation if $S(L) = 2\sqrt{5L}$.

$S(L) = 2\sqrt{5} L^{1/2}$

$S'(L) = \frac{5}{2\sqrt{5}} L^{-1/2}$

$S'(20) = \frac{5}{2\sqrt{20}} = \frac{1}{2} \text{ mph/ft}$

**If car 1 skids 20 ft, and car 2 skids 21 ft, then car 2 was going 1/2 mph more than car 1.**

3. Suppose a filter has been designed to remove 100 grams of sediment from a storage tank. Let $Q(t)$ be the amount of sediment in the tank at time $t$.

**A.** Estimate $Q'(3)$ if the filter removes a fixed amount of sediment each hour, say 2.3 grams.

$Q'(3) = -2.3 \text{ g/hr}$

**During the 4th hour, the filter removes 2.3 grams.**

**B.** Estimate $Q'(3)$ if the filter removes a fixed percentage of sediment each hour, say 20%.

$Q(t) = 100 (t^8)^t$

$Q'(3) = \lim_{h \to 0} \left( 100 (3^h)^8 - 100 (8^h)^8 \right) \frac{t}{h}$

$= -11.425 \text{ g/hr}$

**During the 4th hour, the filter removes 11.425 grams.**

C. Give a practical interpretation of one of your answers above.

**SEE STATEMENTS WITH EACH PROBLEM.**

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4. Estimate \( L'(35) \) and give a practical interpretation. \( L \) is the light output (millions of lumens) and \( t \) is the time after ignition (milliseconds) of a No. 22 lightbulb.

<table>
<thead>
<tr>
<th>Time after ignition</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light output</td>
<td>0</td>
<td>0.2</td>
<td>0.5</td>
<td>2.6</td>
<td>4.2</td>
<td>3.0</td>
<td>1.7</td>
<td>0.7</td>
<td>0.35</td>
<td>0.2</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
\text{Slope from } t=30 \text{ to } t=35: \frac{4.2 - 0.2}{5} = 0.8 \quad \text{(millions of lumens/millisecond)}
\]

\[
\text{Slope from } t=35 \text{ to } t=40: \frac{0.7 - 2.6}{5} = -0.35 \quad \text{millions of lumens/millisecond}
\]

5. The registrar has put a counter on the RSVP registration telephone lines to count the total number of students registering during the day. A graph of \( N(t) \), the total number of students who have registered during the \( t \) hours since noon, is given below.

\[
N(t) = \text{\# of students during } t \text{ hours}
\]

\[
\frac{dN}{dt} = \frac{\text{\# of students}}{\text{per hour}}
\]

A. Estimate \( N^{-1}(2000) \) and give an interpretation.

\[
N^{-1}(2000) = 4.6 \text{ HRS}
\]

The time it takes for 2000 students to register is 4.6 hours.

B. Estimate \( N'(2) \) and give an interpretation.

\[
(1, 0), (3.75, 50)
\]

\[
N'(2) = \frac{950 - 0}{2} = 475 \text{ students/1hr}
\]

Between 2:00 and 3:00 PM, 375 students register.

C. Estimate coordinates of the inflection point. Explain the significance of this point in terms of the problem.

\[
(3.75, 1500) \quad \text{At 3.75 hours after noon, the \# of students registering per hour begins to decrease.}
\]

D. Sketch a graph of \( N'(t) \).

\[
\text{See graph above}
\]